

General Instructions:

1. Question nos. 1 to 6 carry 1 mark each.
2. Question nos. 7 to 19 carry 4 marks each.
3. Question nos. 20 to 26 carry 6 marks each.

SECTION – A

1. Evaluate : $\sin(\cot^{-1}x)$
2. Find the value of k so that matrix $A = \begin{bmatrix} 2 & k \\ 3 & 1 \end{bmatrix}$ has no inverse.
3. For any 2x2 matrix, if $A \cdot (\text{Adj}A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then find the value of $|A|$.
4. Differentiate $\sqrt{4 + \sqrt{4 + x}}$ w.r.t x.
5. Evaluate : $\int 5^{5^x} \cdot 5^x dx$
6. Find the rate of change of volume of sphere with respect to its diameter.

SECTION – B

7. Solve : $\sin^{-1}(1-x) - 2\sin^{-1}x = \pi/2$
8. Simplify : $\tan^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right)$
9. Find the value of k so that function $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ k, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & x > 0 \end{cases}$

is continuous at $x=0$.

10. Show that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$
11. If $y = (\tan x)^{\log x} + \cos^2\left(\frac{\pi}{4}\right)$ find $\frac{dy}{dx}$
12. Find inverse of matrix $A = \begin{bmatrix} 0 & 3 \\ 5 & 2 \end{bmatrix}$ using elementary transformation method.

13. Express $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$ as sum of symmetric and skew symmetric matrix.
14. Evaluate $\int \frac{dx}{\cos(x + \alpha) \sin(x + \beta)}$
15. Evaluate $\int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-x/2} dx$
16. Show that the curves $4x = y^2$ and $4xy = k$ cut at right angle if $k^2 = 512$.
17. Find the intervals for which $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is increasing or decreasing.
18. Find the approximate value of $\sqrt{0.037}$ using differential approximations.
19. Evaluate : $\int \frac{x^2}{(x-1)(x-2)} dx$

SECTION – C

20. Solve using matrix method
 $x - y + z = 1$; $2x + y - z = 2$; $2x + y - 3z = 0$
21. Let $A = \begin{bmatrix} 0 & -\tan \alpha/2 \\ \tan \alpha/2 & 0 \end{bmatrix}$ and I be identity matrix of order 2. Show that
 $I+A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$
22. Evaluate $\int \sqrt{\tan x} dx$
23. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half of that cone.
24. Find the equation of tangent line to the curve $y = x^2 - 2x + 7$ which is
 a) Parallel to line $2x - y + 9 = 0$
 b) Perpendicular to line $5y - 15x = 13$
25. Evaluate $\int \frac{dx}{\sin x - \sin 2x}$
26. If $\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2 - y^2)$ prove that $\frac{dy}{dx} = \frac{x}{y} \sqrt{\frac{1-y^4}{1-x^4}}$