

General Instructions:

1. Question nos. 1 to 4 carry 1 mark each.
2. Question nos. 5 to 11 carry 4 marks each.
3. Question nos. 12 to 14 carry 6 marks each.

**SECTION - A**

1. If  $\int_0^1 (3x^2 + 2x + k)dx = 0$ , find the value of k.
2. Find the value of p, if  $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$
3. Write the direction cosines of a line parallel to z-axis.
4. Let \* be a binary operation on N given by  $a*b = \text{H.C.F}(a,b) \forall a, b \in \mathbb{N}$ . Write the value of  $22 * 4$ .

**SECTION - B**

5. Evaluate  $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$
6. Solve :  $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$ .
7. Solve :  $(3xy + y^2)dx + (x^2 + xy)dy = 0$ .
8. The scalar product of vector  $\hat{i} + 2\hat{j} + 4\hat{k}$  with a unit vector along the sum of vectors  $\hat{i} + 2\hat{j} + 3\hat{k}$  and  $\lambda\hat{i} + 4\hat{j} - 5\hat{k}$  is equal to one. Find the value of  $\lambda$ .
9. Find the distance of the point  $(-2, 3, -4)$  from the line  $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$  measured parallel to plane  $4x + 12y - 3z + 1 = 0$ .
10. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.
11. Let  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{1\}$ . Consider the function  $f : A \rightarrow B$  defined by  $f(x) = \frac{x-2}{x-3}$ . Show that f is bijective and hence find  $f^{-1}$ .

**SECTION - C**

12. Using integration find the area bounded by  $\{(x, y) : x^2 + y^2 \leq 8x, y^2 \geq 4x, x \geq 0, y \geq 0\}$
13. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the minimum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as linear programming problem and solve graphically.
14. A letter is known to have either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter has come from CLIFTON.